Formule de trigonometrie

$\sin^2 x + \cos^2 x = 1$ formula fundamentală a trigonometriei

$\sin : \mathbb{R} \to [-1,1]$

$\sin(-x) = -\sin x$ funcţia sin este impară

$\cos : \mathbb{R} \to [-1,1]$

$\cos(-x) = \cos x$ funcţia cos este pară

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$tg(-x) = -tgx$

$ctg(-x) = -ctgx$

$$\sin 2x = 2\sin x \cos x \quad \Rightarrow \quad \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 2\cos^2 x - 1 \quad \Rightarrow \quad \cos^2 x = \frac{1+\cos 2x}{2}$$

$$\cos 2x = 1 - 2\sin^2 x \quad \Rightarrow \quad \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\sin 3x = \sin x(3 - 4\sin^2 x)$$

$$\cos 3x = \cos x(4\cos^2 x - 3)$$

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$\sin(a-b) = \sin a \cos b - \sin b \cos a$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$tg(a+b) = \frac{tga + tgb}{1 - tga \cdot tgb}$$

$$tg(a-b) = \frac{tga - tgb}{1 + tga \cdot tgb}$$

$$tgx = \frac{\sin x}{\cos x}$$

$$ctgx = \frac{\cos x}{\sin x}$$

Formule pentru transformarea sumelor in produse

$$\sin p + \sin q = 2\sin\frac{p+q}{2}\cos\frac{p-q}{2}$$

$$\sin p - \sin q = 2\sin\frac{p-q}{2}\cos\frac{p+q}{2}$$

$$\cos p + \cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$$

$$\cos p - \cos q = -2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$$

Formule pentru transformarea produselor in sume

$$\sin x \cdot \cos y = \frac{\sin(x+y) + \sin(x-y)}{2}$$

$$\cos x \cdot \cos y = \frac{\cos(x+y) + \cos(x-y)}{2}$$

$$\sin x \cdot \sin y = \frac{\cos(x-y) - \cos(x+y)}{2}$$

$$tg3x = \frac{3tgx - tg^3x}{1 - 3tg^2x}$$

$$ctg3x = \frac{ctg^3x - 3ctgx}{3ctg^2x - 1}$$

$$\begin{cases} \sin x = \dfrac{2t}{1+t^2} \\ \cos x = \dfrac{1-t^2}{1+t^2} \\ tgx = \dfrac{2t}{1-t^2} \\ ctgx = \dfrac{1-t^2}{2t} \end{cases} \quad \text{unde } t = tg\frac{x}{2}$$

$$\begin{cases} \sin 2x = \dfrac{2tgx}{1+tg^2x} \\ \cos 2x = \dfrac{1-tg^2x}{1+tg^2x} \\ tg2x = \dfrac{2tgx}{1-tg^2x} \\ ctg2x = \dfrac{1-tg^2x}{2tgx} \end{cases}$$

Ecuaţii trigonometrice fundamentale

1)Ecuaţia $\sin x = a$ are soluţii dacă şi numai dacă $a \in [-1,1]$.
In acest caz soluţiile sunt

$$x \in \{(-1)^k \arcsin a + k\pi \,/\, k \in \mathbb{Z}\}.$$

2)Ecuaţia $\cos x = b$ are soluţii dacă şi numai dacă $b \in [-1,1]$.
In acest caz soluţiile sunt

$$x \in \{\pm \arccos b + 2k\pi \,/\, k \in \mathbb{Z}\}.$$

3)Ecuaţia $tgx = c$ are soluţii $\forall c \in \mathbb{R}$.
Soluţiile sunt $x \in \{\mathrm{arc}\,tgc + k\pi \,/\, k \in \mathbb{Z}\}$.

4)Ecuaţia $ctgx = d$ are soluţii $\forall d \in \mathbb{R}$.
Soluţiile sunt $x \in \{\mathrm{arc}\,ctgd + k\pi \,/\, k \in \mathbb{Z}\}$.

$$\left.\begin{array}{l} \sin(\arcsin x) = x \\ \sin(\arccos x) = \sqrt{1-x^2} \\ \cos(\arccos x) = x \\ \cos(\arcsin x) = \sqrt{1-x^2} \end{array}\right\} \forall x \in [-1.1]$$